Microwaves

Series 2

Problem 1

A pulsed RADAR works at a frequency of 10 GHz. Its antenna has a gain of 28 dB, and the emitted pulsed power is of 2 kW. The target of the RADAR has an equivalent radar surface of 12 m^2, and the detector has a sensitivity of –90 dBm. What is the distance range of the RADAR for this target?

$$G = 10^{\frac{28}{10}} = 631$$

$$P_{\min} = 10^{\frac{-90}{10}} [`mW] = 10^{-12} [W]$$

$$\lambda = 0.03 [m]$$

$$R_{\max} = \left[\frac{\left(2 \cdot 10^3\right)\left(631\right)^2 \left(12\right)\left(0.03\right)^2}{\left(4\pi\right)^3 \left(10^{-12}\right)} \right]^{\frac{1}{4}} = 8114 \ [m]$$

Problem 2

Two sailboats, located at the equator and at 30° and 31° west respectively are communicating in Short-wave (3 MHz). The transceiver of Boat A emits at a power of 100W. What power will the transceiver of boat B receive? We suppose that the antennas used are isotropic and that the attenuation at 3 MHz at sea level is of 0.01 dB/km

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The distance between the sailboats is given by

$$l = 2\sin\left(0.5°\right)R' = 148km$$

The wavelength at 3 MHz is 100m. The antennas have a gain of 1, so Friis formula gives

$$\frac{P_r}{P_e} = g_r g_e \left(\frac{\lambda}{4\pi d}\right)^2 = 2.9 \cdot 10^{-9}$$

which corresponds to an attenuation of 85 dB. The attenuation due to the losses in the atmosphere is of 148*0.01=1.5dB, yielding a total attenuation of 86.5 dB, corresponding to à 2.24 10^{-9}. The emitted power being 100W, the received power will be

$$P_r = P_e \cdot 2.24 \cdot 10^{-9} = 0.22\mu W$$

Problem 3

Let us consider a wireless point to point transmission at 4 GHz, covering a distance of 150km over a flat terrain. At which minimal height over the terrain should the antennas be placed? We consider two different cases:

a) Both antennas are at the same level above the ground.

b) The first antenna is at an altitude of 100m over the ground. At which height should the second be?

The radius of the Earth is much larger than the altitude h1 and h2 of the antennas above the ground. The problem, which is a geometrical one, can thus be simplified by assuming that the height of the antennas is given by the axes of the Fresnel ellipsoid added to the eight due to the curvature of the Earth:

(h1=ρ+y1, h2=ρ+y2, c.f. figure).

L1 is the distance between the first antenna and the point where the Fresnel ellipsoid is tangential to the Earth, while L2 is the distance between this point and the second antenna. We have thus:

$$h1 = \rho + y1$$
$$h2 = \rho + y2$$
$$L = L1 + L2 = 150km$$
$$\rho = 0.5\sqrt{\lambda L} = 53m$$

We now obtain y1 and y2 using geometrical considerations: Pythagoras theorem allows us to write:

$$(R' + yi)^2 = R'^2 + Li^2 \quad i = 1,2$$
$$R'^2 + 2R'yi + yi^2 = R'^2 + Li^2$$

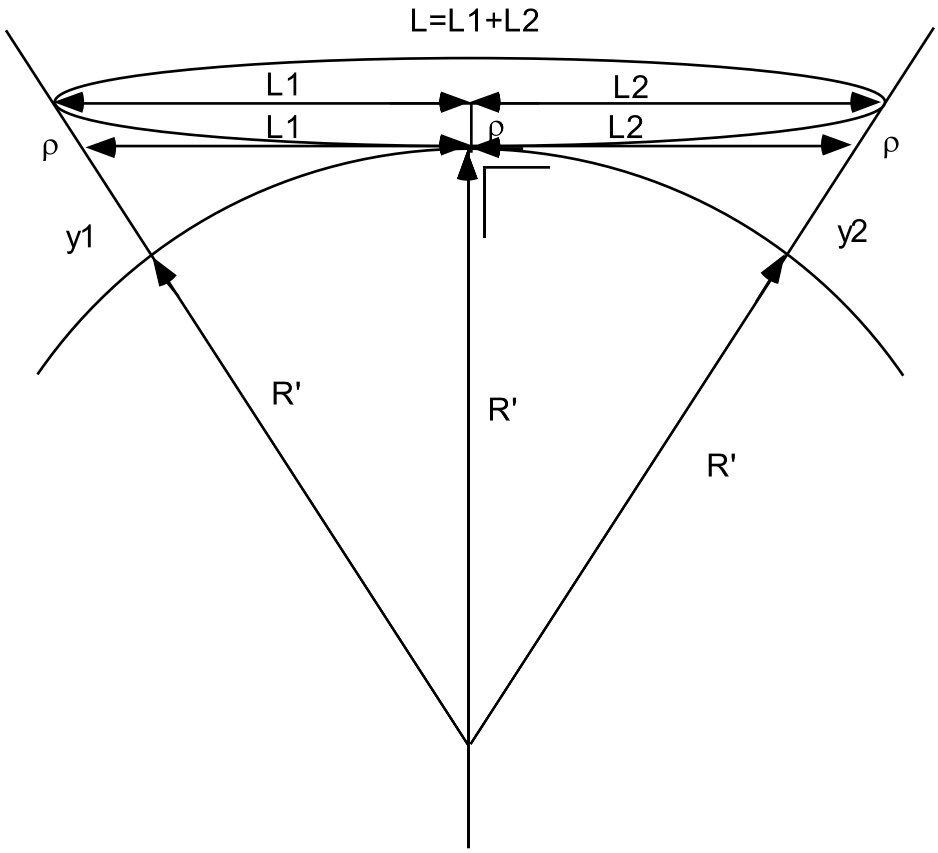
$$yi \ll R' \text{thus } yi^2 \text{ is negligible and}$$

$$yi = \frac{Li^2}{2R'}$$

a) L1=l2=75km, thus y1=y2=330m and h1=h2=383m

b) h1=100m thus y1=47m, L1=28km, L2=122km, y2=876m and h2=929m.

We see here the usefulness of using hills and mountains to place point to point link antennas.



Problem 4
Find the phasor corresponding to the following field :

$$\mathbf{E(t)} = 32\mathbf{e_x}\sin(\omega t + \pi/8) + 27\mathbf{e_y}\sin(\omega t + 3\pi/8)$$

Find its real and imaginary parts, and verify that

$$\mathrm{Re}[\underline{\mathbf{E}}] = \mathbf{E}(t=0), \ \mathrm{Im}[\underline{\mathbf{E}}] = -\mathbf{E}(t = T/4)$$

The cosine is the real part of a complex exponential. It is thus useful to transform the expression of the field into

$$\begin{aligned}\mathbf{E}(t) &= 32\mathbf{e_x}\sin(\omega t + \pi/8) + 27\mathbf{e}_y\sin(\omega t + 3\pi/8)\\ &= 32\mathbf{e_x}\cos(\omega t - 3\pi/8) + 27\mathbf{e_y}\cos(\omega t - \pi/8)\\ &= \mathrm{Re}\left[\left(32\mathbf{e_x}e^{-j3\pi/8} + 27\mathbf{e_y}e^{-j\pi/8}\right)e^{j\omega t}\right]\end{aligned}$$

were we used sin(t) = cos(t-π/2)
From this, dividing by $\sqrt{2}$, the phasor is written as :

$$\mathbf{E} = 22.63\mathbf{e_x}e^{-j3\pi/8} + 19.09\mathbf{e_y}e^{-j\pi/8}$$

It is now easy to verify that :

$$\mathrm{Re}\left[\mathbf{E}\right] = 22.63\mathbf{e_x}\cos(3\pi/8) + 19.09\mathbf{e_y}\cos(\pi/8) = 8.66\mathbf{e_x} + 17.64\mathbf{e_y}$$
$$\mathrm{Im}\left[\mathbf{E}\right] = -22.63\mathbf{e_x}\sin(3\pi/8) - 19.09\mathbf{e_y}\sin(\pi/8) = -20.91\mathbf{e_x} - 7.31\mathbf{e_y}$$

$$\mathbf{E}(\mathbf{0}) = 32\mathbf{e_x}\sin(\pi/8) + 27\mathbf{e_y}\sin(3\pi/8) = 12.246\mathbf{e_x} + 24.95\mathbf{e}_y = \sqrt{2}\,\mathrm{Re}\left[\mathbf{E}\right]$$
$$\mathbf{E}(T/4) = 32\mathbf{e_x}\sin(\pi/2 + \pi/8) + 27\mathbf{e_y}\sin(\pi/2 + 3\pi/8) = 29.56\mathbf{e_x} + 10.33\mathbf{e}_y = -\sqrt{2}\,\mathrm{Im}\left[\mathbf{E}\right]$$